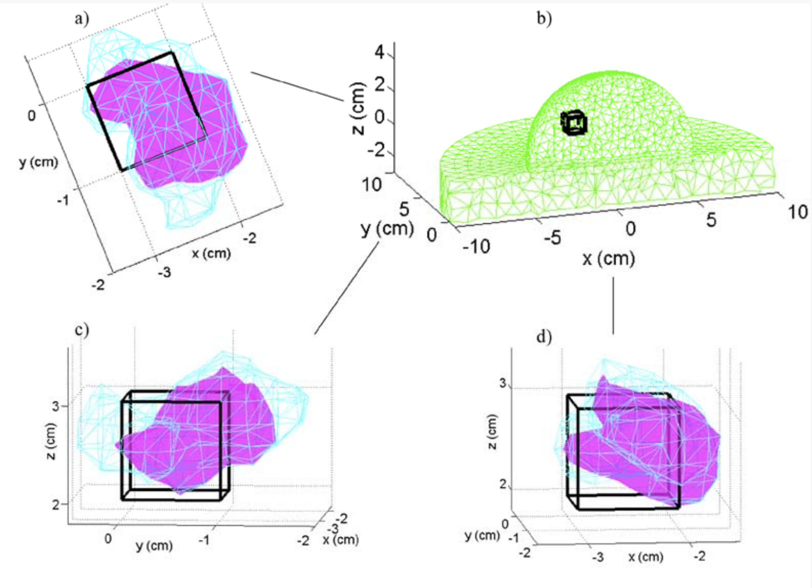
FEM/BEM-based Fluorescence Optical Tomography for Breast Cancer Diagnostic Imaging



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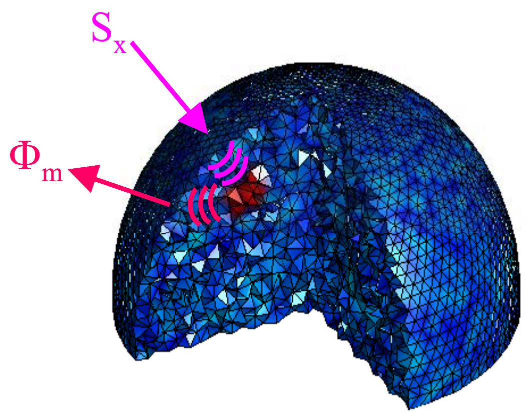
FREQUENCY-DOMAIN PHOTON MIGRATION PDE's

$$-\nabla \bullet \left(D_x \nabla \Phi_x\right) + k_x \Phi_x = S_x$$

$$-\nabla \bullet \left(D_m \nabla \Phi_m\right) + k_m \Phi_m = \beta \Phi_x$$



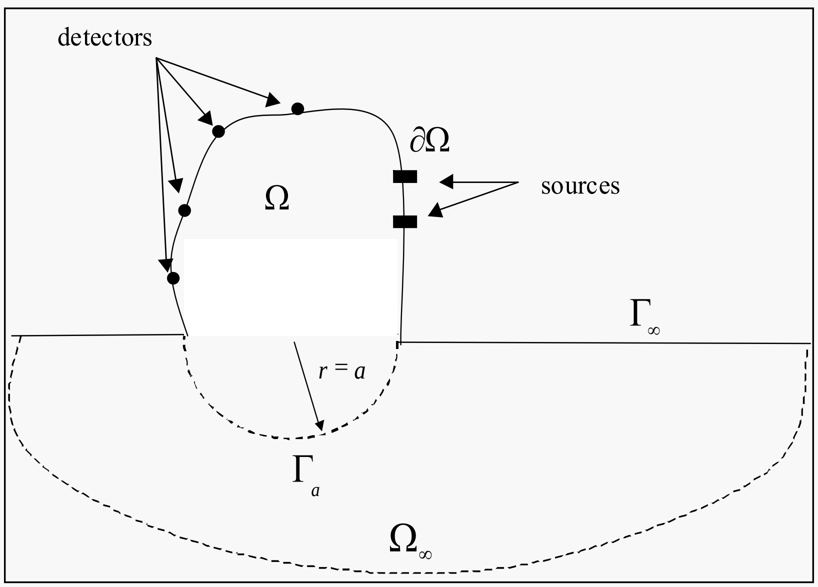
$$\begin{cases} D_x \dfrac{\partial \Phi_x}{\partial n} + r_x \Phi_x = 0 \\ \qquad\qquad\qquad\qquad on \quad \partial\Omega \\ D_m \dfrac{\partial \Phi_m}{\partial n} + r_m \Phi_m = 0 \end{cases}$$

THE INVERSE PROBLEM

$$L\Phi(x,p) = f(p) \qquad \text{L non-selfadjoint operator}$$

$$F(p) = \sum_{n=1}^{N} \left(\Phi_n^{det} - \Phi_{meas}^{det}\right)^2$$

$$find\ \ p \quad such\ \ that \quad F(p)\ \ \text{min}$$



Sensitivity key term !

$$\frac{\delta F}{\delta p} = 2\sum_{n=1}^{N}\left(\Phi_n^{\ det} - \Phi_{meas}^{\ det}\right)\frac{\delta \Phi_n^{\ det}}{\delta p} = 0$$

$$\frac{\underline{\mathbf{\Phi}}(\underline{x}_{det}, p+\delta p) - \underline{\mathbf{\Phi}}(\underline{x}_{det}, p)}{\delta p}$$

Computationally impractical !!

COMPLETE PERTURBATION EQUATIONS

Perturbation parameter

$$p \to p + \delta p \quad \Rightarrow \quad \underline{\mathbf{\Phi}} \to \underline{\mathbf{\Phi}} + \underline{\delta\mathbf{\Phi}}$$

Perturbation equations

Matrix notation of coupled equations

$$-\underline{\underline{\nabla}}^t\left(\underline{\underline{\mathbf{d}}}\,\underline{\underline{\nabla}}\,\underline{\delta\mathbf{\Phi}}\right)+\underline{\underline{\mathbf{k}}}\,\underline{\delta\mathbf{\Phi}}=\underline{\underline{\nabla}}^t\left(\frac{\partial\underline{\underline{\mathbf{d}}}}{\partial p}\delta p\,\underline{\underline{\nabla}}\,\underline{\mathbf{\Phi}}\right)-\frac{\partial\underline{\underline{\mathbf{k}}}}{\partial p}\delta p\,\underline{\underline{\mathbf{\Phi}}} \quad \text{on } \Omega$$

$$\underline{\underline{\mathbf{D}}}\frac{\partial\underline{\delta\mathbf{\Phi}}}{\partial n}+\underline{\underline{\mathbf{r}}}\,\underline{\delta\mathbf{\Phi}}=-\left(\frac{\partial\underline{\underline{\mathbf{D}}}}{\partial p}\delta p\frac{\partial\underline{\mathbf{\Phi}}}{\partial n}+\frac{\partial\underline{\underline{\mathbf{r}}}}{\partial p}\delta p\,\underline{\mathbf{\Phi}}\right) \quad \text{on } \partial\Omega$$

We are interested in $\underline{\delta\mathbf{\Phi}}$ At the detectors

ADJOINT EQUATIONS

Multiply by an arbitrary matrix $\underline{\underline{\Psi}}^t$

$$\int_{\Omega} \underline{\underline{\Psi}}^t\left(-\underline{\underline{\nabla}}^t\left(\underline{\underline{\mathbf{d}}}\,\underline{\underline{\nabla}}\,\delta\underline{\mathbf{\Phi}}\right)+\underline{\underline{\mathbf{k}}}\,\delta\underline{\mathbf{\Phi}}\right)d\Omega = \int_{\Omega} \underline{\underline{\Psi}}^t\left(\underline{\underline{\nabla}}^t\left(\frac{\partial \underline{\underline{\mathbf{d}}}}{\partial p}\delta p\,\underline{\underline{\nabla}}\,\underline{\mathbf{\Phi}}\right)-\frac{\partial \underline{\underline{\mathbf{k}}}}{\partial p}\delta p\,\underline{\mathbf{\Phi}}\right)d\Omega$$

Integration by parts twice & applying boundary conditions

$$\delta\underline{\mathbf{\Phi}}(\underline{\mathbf{x}}_{\det}) = \int_{\Omega} \underline{\underline{\Psi}}^t(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det})\left(\underline{\underline{\nabla}}^t\left(\frac{\partial \underline{\underline{\mathbf{d}}}}{\partial p}\delta p\,\underline{\underline{\nabla}}\,\underline{\mathbf{\Phi}}\right)-\frac{\partial \underline{\underline{\mathbf{k}}}}{\partial p}\delta p\,\underline{\mathbf{\Phi}}\right)d\Omega + \int_{\partial\Omega} \underline{\underline{\Psi}}^t(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det})\left(-\frac{\partial \underline{\underline{\mathbf{D}}}}{\partial p}\delta p\frac{\partial \underline{\mathbf{\Phi}}}{\partial n}-\frac{\partial \underline{\underline{\mathbf{r}}}}{\partial p}\delta p\,\underline{\mathbf{\Phi}}\right)dS$$

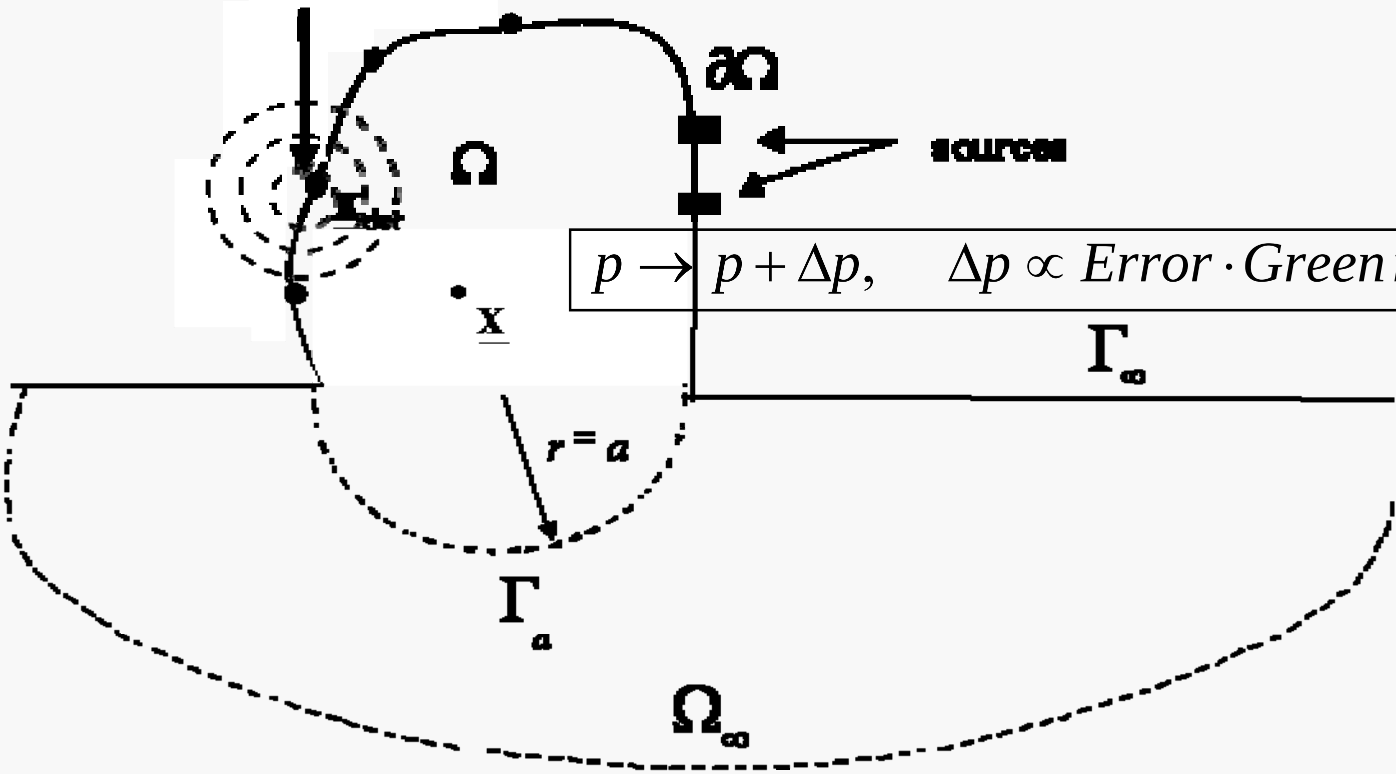
$\underline{\underline{\Psi}}$ is ***"Green matrix"*** for the detector locations $\underline{\mathbf{x}}_{\det}$

$$\underline{\underline{\Psi}}(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det}) = \begin{bmatrix} \mathbf{\Psi}_{xx}(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det}) & \mathbf{\Psi}_{xm}(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det}) \\ \mathbf{\Psi}_{mx}(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det}) & \mathbf{\Psi}_{mm}(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det}) \end{bmatrix}$$

$$\begin{cases} -\underline{\underline{\nabla}}^t\left(\underline{\underline{\mathbf{d}}}\,\underline{\underline{\nabla}}\,\underline{\underline{\Psi}}\right)+\underline{\underline{\mathbf{k}}}^t\underline{\underline{\Psi}} = \underline{\underline{\boldsymbol{\delta}}}(\underline{\mathbf{x}};\underline{\mathbf{x}}_{\det}) \text{ on } \Omega \\ \underline{\underline{\mathbf{D}}}\dfrac{\partial \mathbf{\Psi}}{\partial n}+\underline{\underline{\mathbf{r}}}\,\underline{\underline{\Psi}} = 0 \text{ on } \partial\Omega. \end{cases}$$

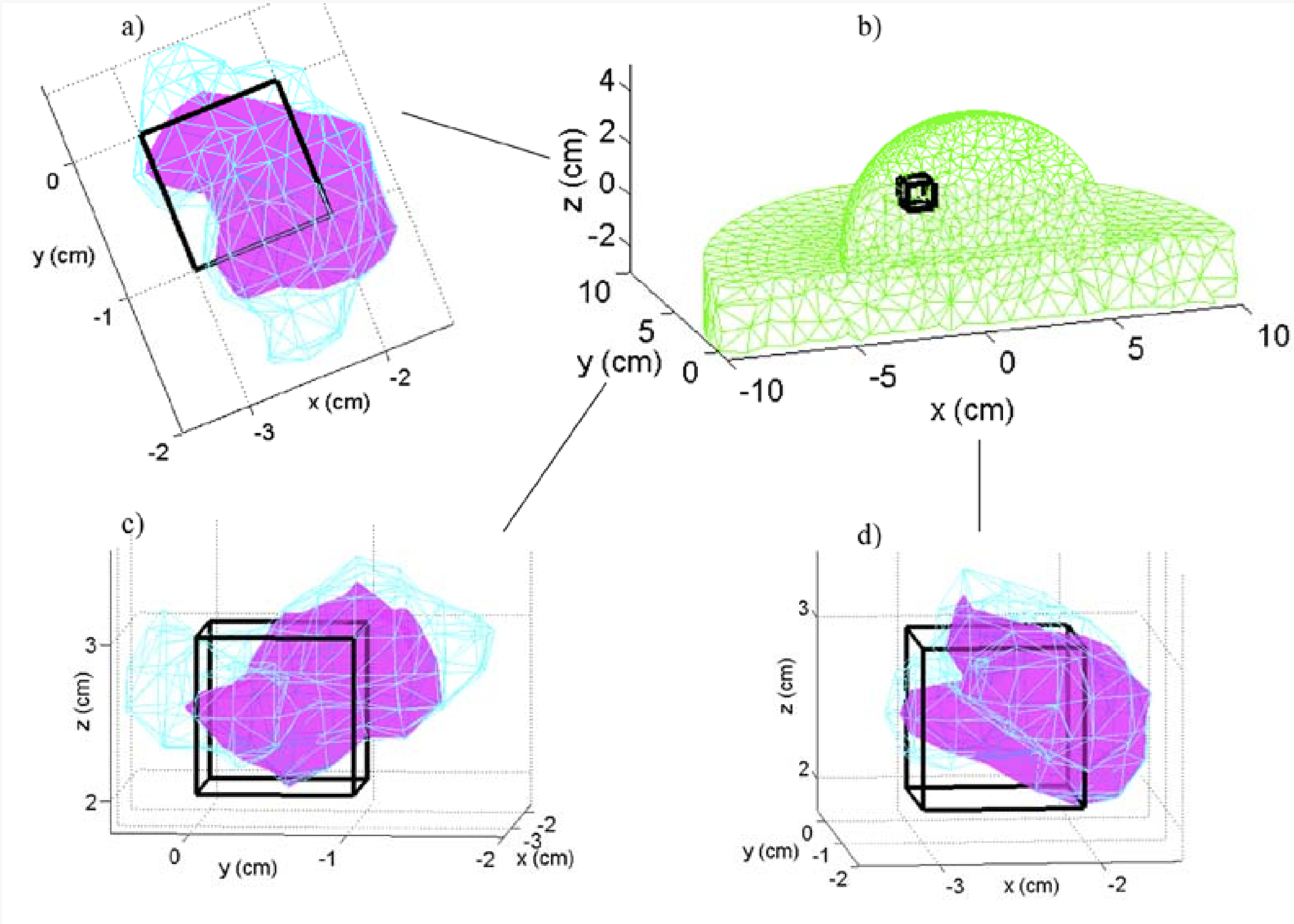
INVERSE ALGORITHM & THE GREEN MATRIX

$$error = \Phi_{data} - \Phi(p)$$



$$p \rightarrow p + \Delta p, \quad \Delta p \propto Error \cdot Green\,matrix$$

FINITE ELEMENT RECONSTRUCTION



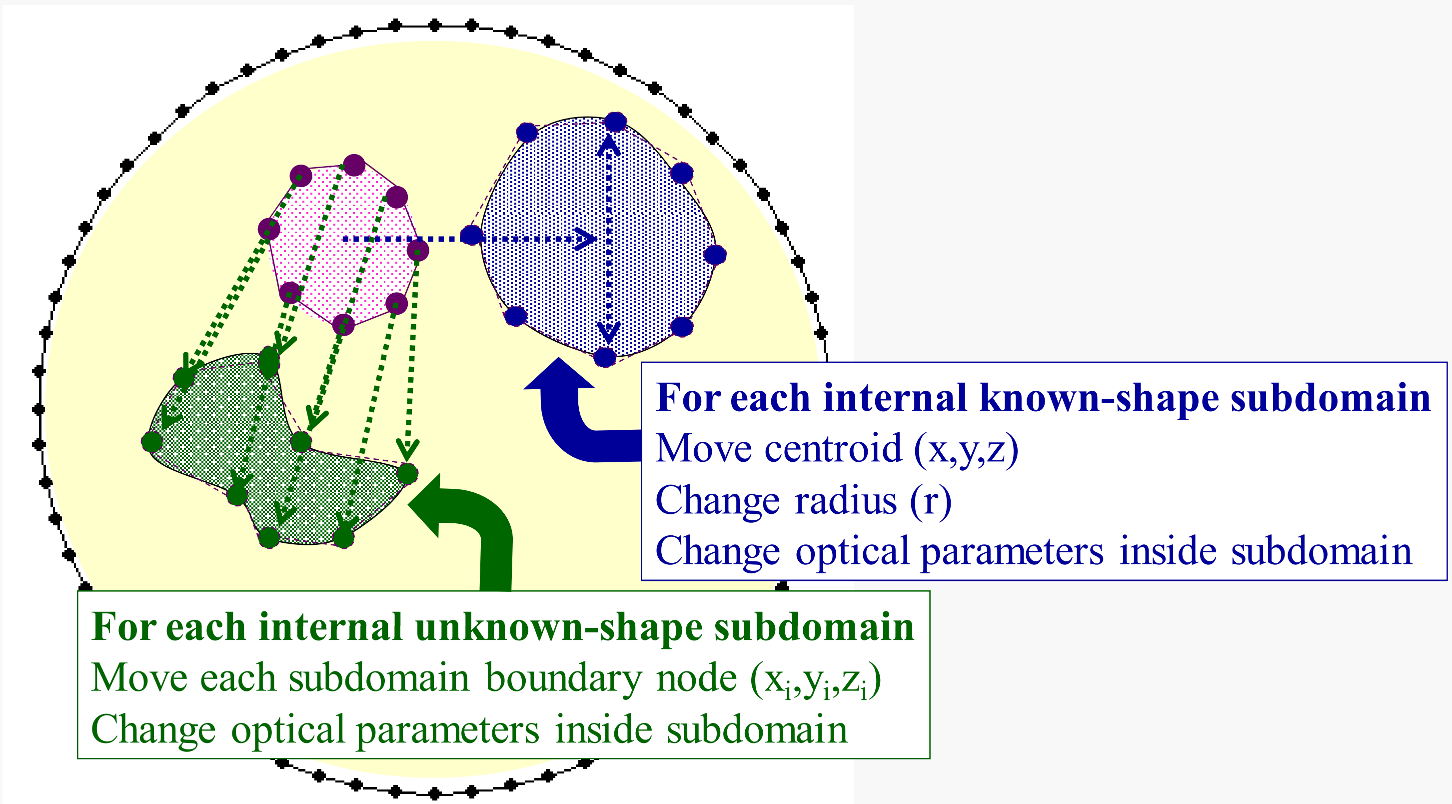
BEM: INTEGRAL FORMULATION

$\underline{\underline{\Psi}}$ ***"Green matrix in infinite space"***

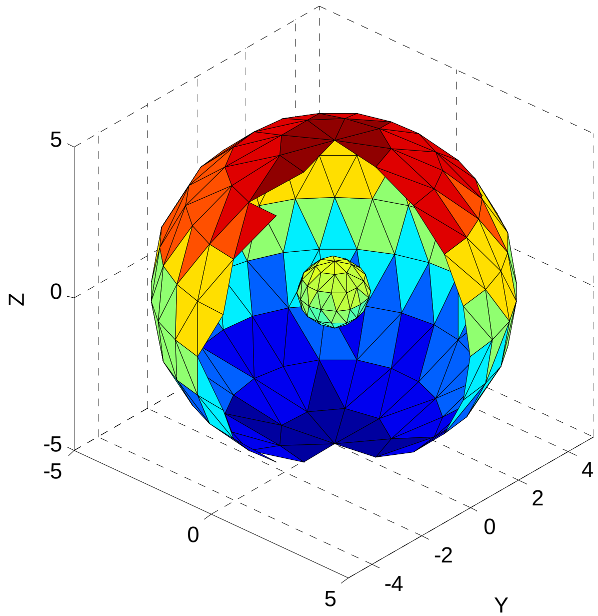
$$\underline{\boldsymbol{\Phi}}(\underline{\boldsymbol{x}}_0) + \int_{\partial\Omega} \left(-\underline{\underline{\boldsymbol{\Psi}}}^t \frac{\partial \underline{\boldsymbol{\Phi}}}{\partial \boldsymbol{n}} + \frac{\partial \underline{\underline{\boldsymbol{\Psi}}}}{\partial \boldsymbol{n}} \underline{\boldsymbol{\Phi}} \right) d\boldsymbol{S} = \int_{\Omega} \underline{\underline{\boldsymbol{\Psi}}}^t \underline{\underline{\mathbf{d}}}^{-1} \underline{\boldsymbol{S}} \, d\Omega$$

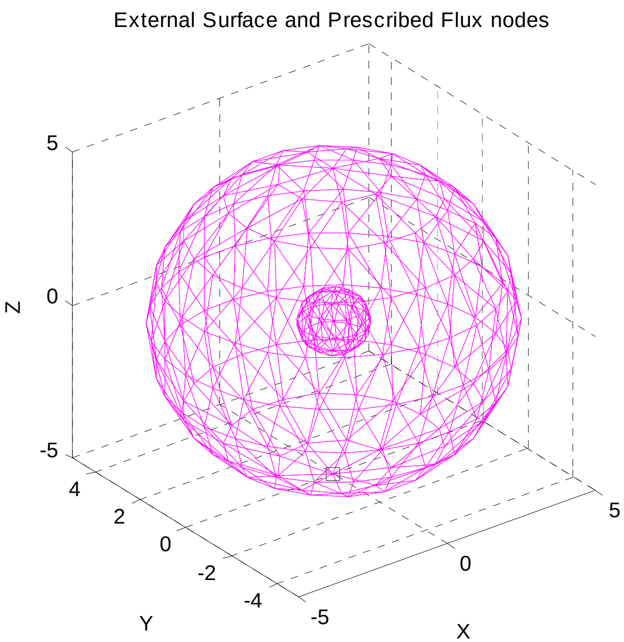
$$\underline{\underline{\boldsymbol{\Psi}}}(\underline{\boldsymbol{x}}, \underline{\boldsymbol{x}}_0) = \begin{bmatrix} \dfrac{\exp(-\boldsymbol{i}\lambda_1 |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|)}{4\pi |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|} & 0 \\ \boldsymbol{\alpha} \left(\dfrac{\exp(-\boldsymbol{i}\lambda_2 |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|)}{4\pi |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|} - \dfrac{\exp(-\boldsymbol{i}\lambda_1 |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|)}{4\pi |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|} \right) & \dfrac{\exp(-\boldsymbol{i}\lambda_2 |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|)}{4\pi |\underline{\boldsymbol{x}} - \underline{\boldsymbol{x}}_0|} \end{bmatrix}$$

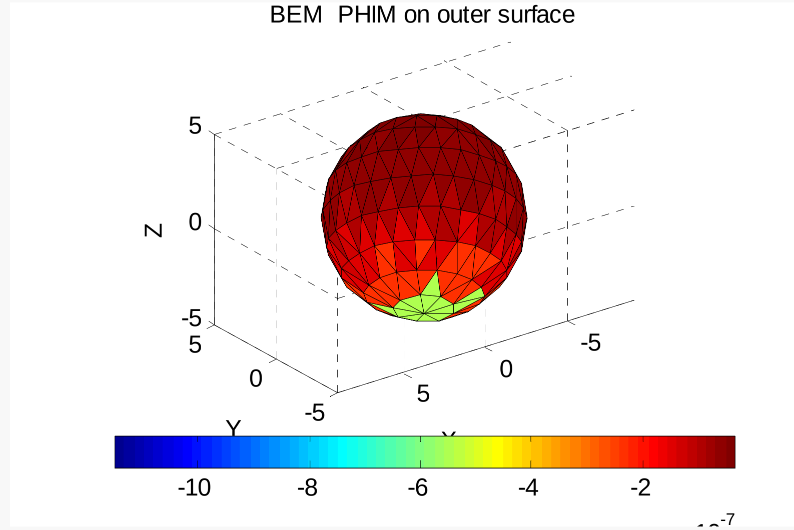
Potential advantage of BEM in fluorescence tomography

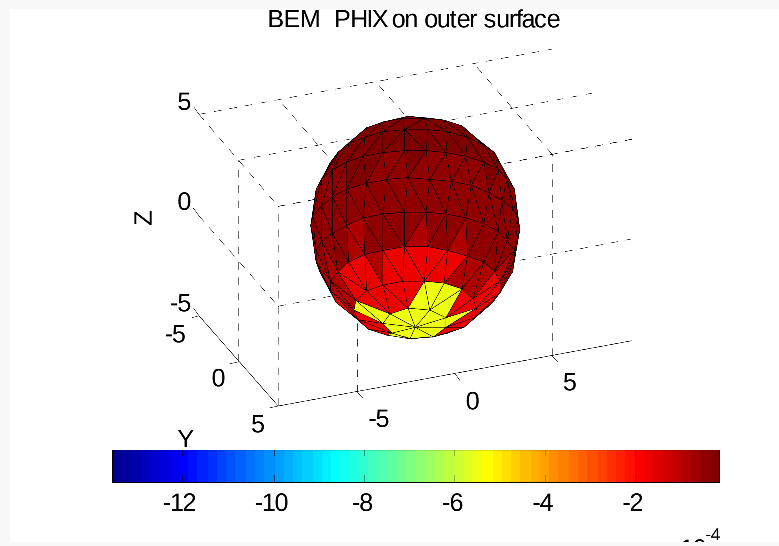


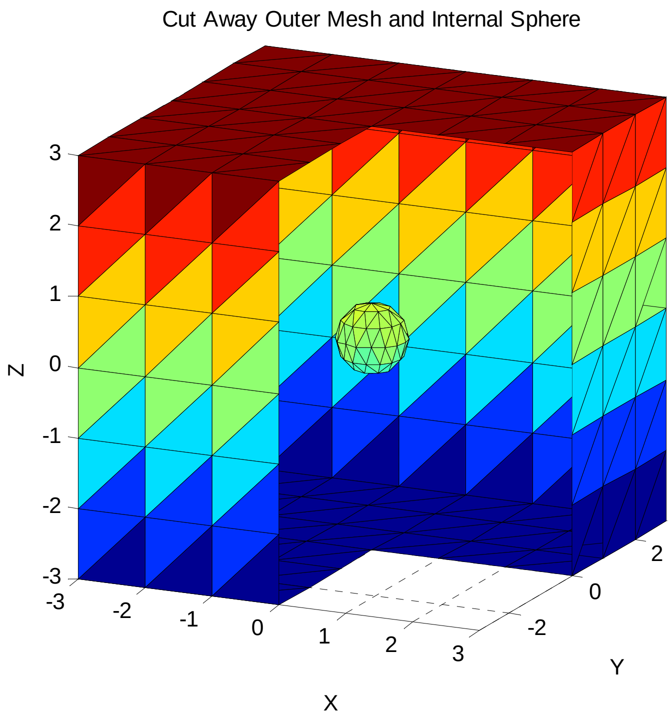
SOME APPLICATIONS

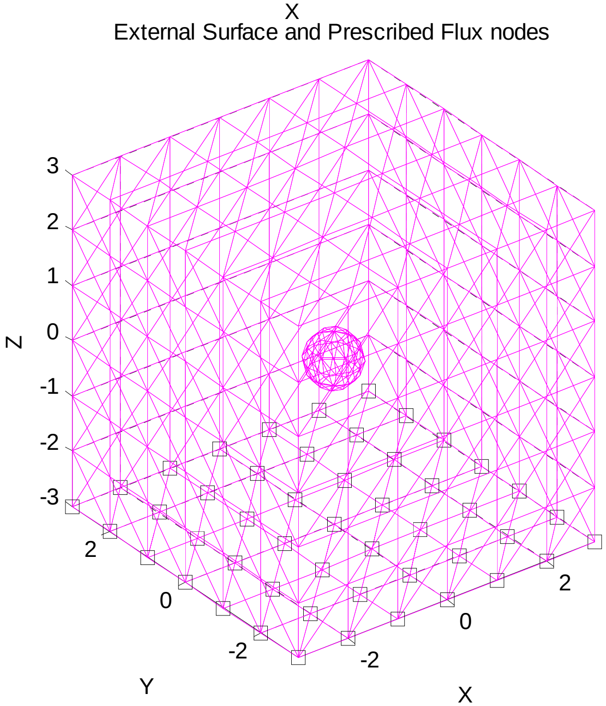


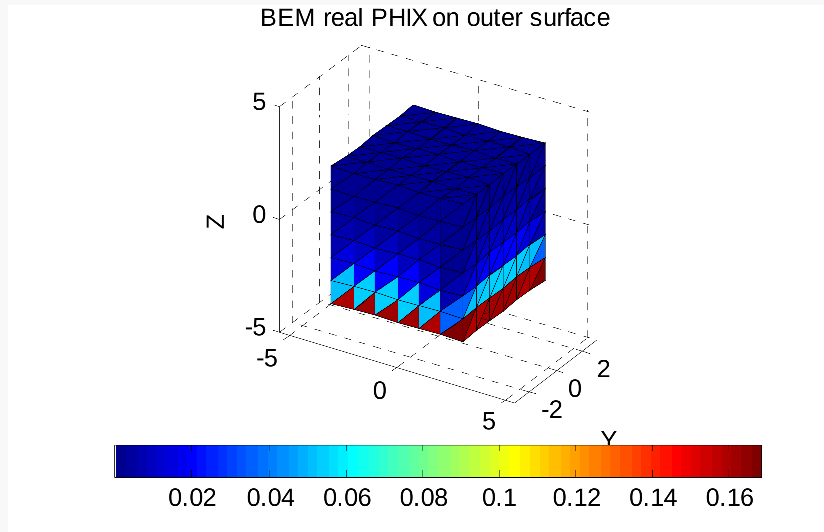


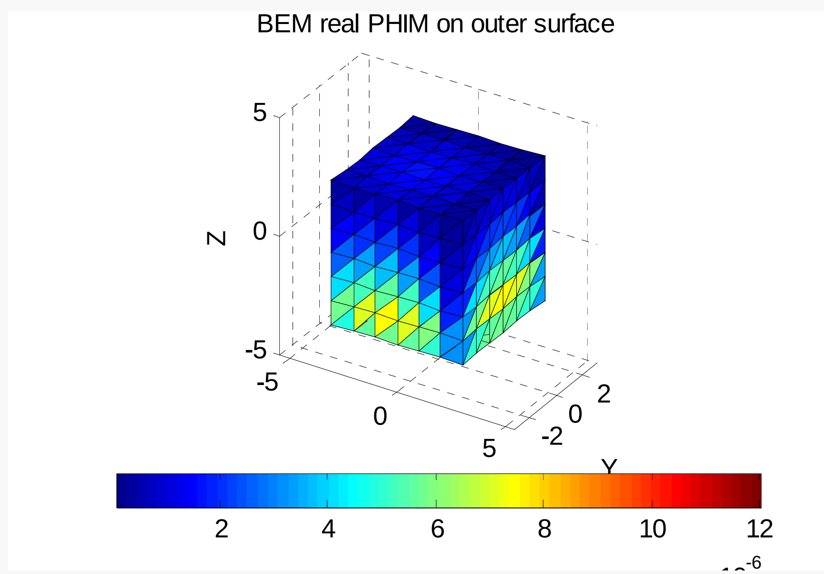




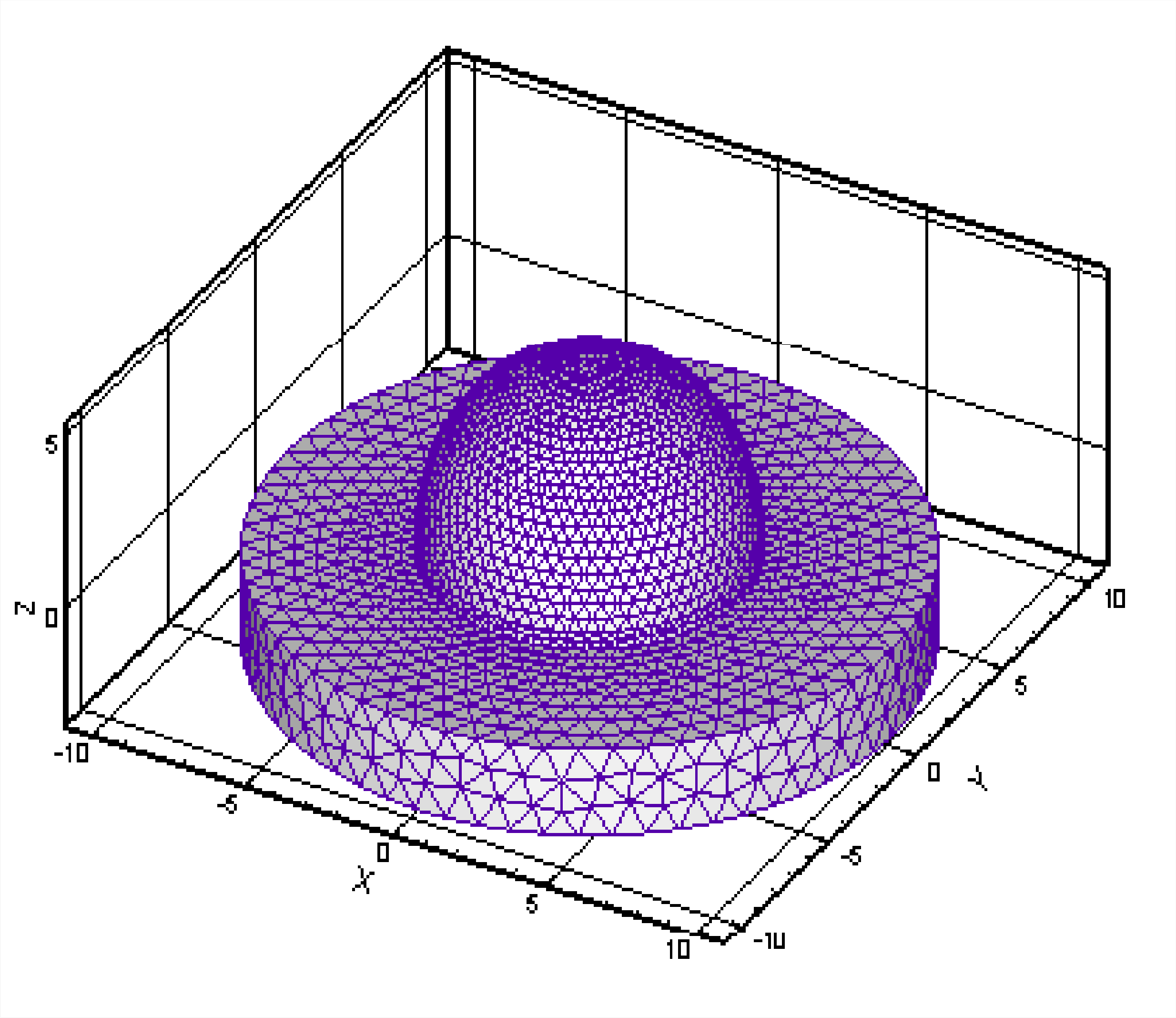

Cut Away Outer Mesh and Internal Sphere
Z
X
Y
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3
2
0
-2


External Surface and Prescribed Flux nodes
Z
Y
X
3
2
1
0
-1
-2
-3
2
0
-2
2
0
-2


BEM real PHIX on outer surface
Z
Y
5
0
-5
-5
0
5
2
0
-2
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16

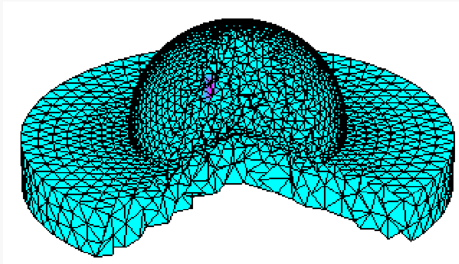

BEM real PHIM on outer surface
Z
Y
5
0
-5
-5
0
5
2
0
-2
2
4
6
8
10
12

BREAST-PHANTOM APPLICATIONS

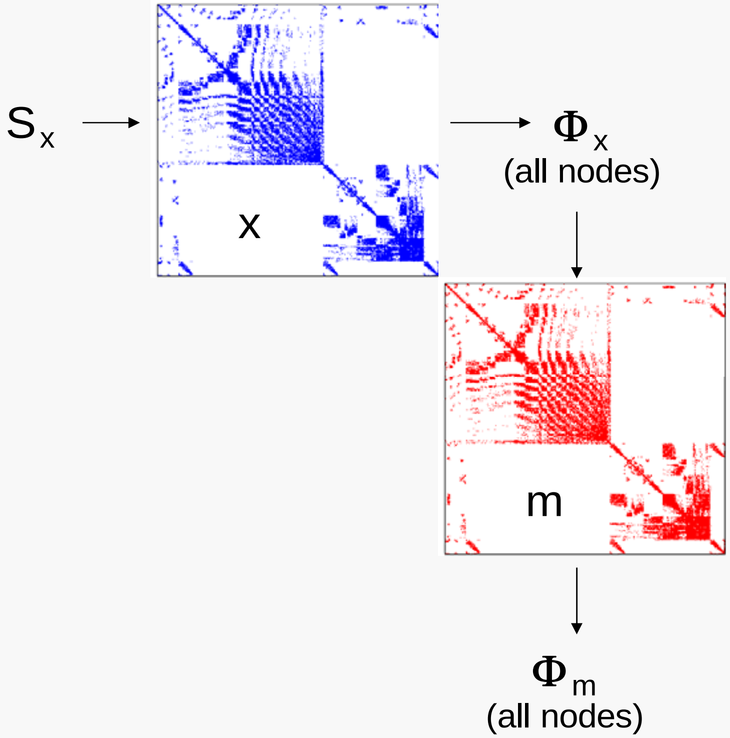


FEM vs. BEM: Mesh and System Matrices

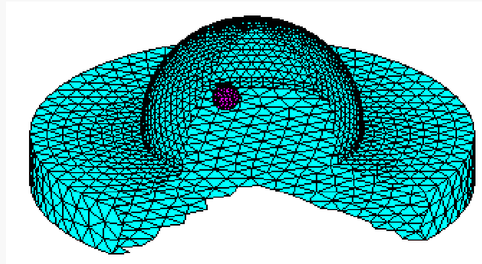
FEM



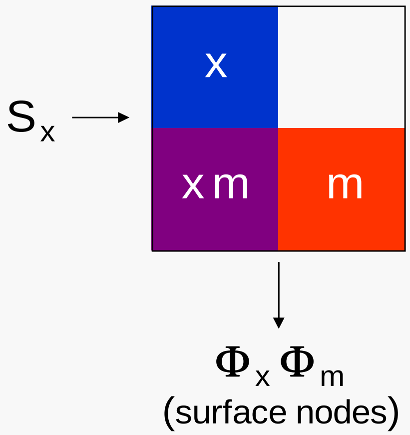
Requires internal mesh. Poor resolution of discrete internal targets.



BEM



No internal mesh required. High resolution of discrete internal targets.



FEM vs BEM: accuracy